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Final Exam MTH 221 , Fall 2011

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LECTURE TIME:——————————————-

QUESTION 1. (Each = 3 points, Total = 69 points) Circle the correct letter.

(i) Let $A = \begin{bmatrix} a_1 & 2 & 0 \\ a_2 & a_3 & 2 \\ a_4 & -4 & 0 \end{bmatrix}$ such that det(A) = -6. The value of x_1 in solving the system $AX = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

a) 1/6 b) -1/6 c) -2 d) -1/3 e) None of the previous.

(ii) Let $T : R^2 \to R$ be a linear transformation such that $T(0,4) = 12$ and $T(2,-1) = 7$. Then $T(4,0) =$

a) -12 b) 12 c) 20 d) 5 e) -5

(iii) Given $D = \{(a,b,c) \in R^3 \mid a+b = 0$ and $a+c = 0\}$ is a subspace of R^3. Then $D =$

a) $span\{(1,0,-1),(1,-1,0)\}$ b) $span\{(-3,3,3)\}$ c) $span\{(0,1,-1),(1,-1,0)\}$ d) None is correct

(iv) One of the following is true:

a) $\{A \in R_{2\times2} \mid det(A) = 0\}$ is a subspace of $R_{2\times2}$ b) $\{(a,b,c) \in R^3 \mid a,b,c \in R \ and \ \ a+b+c = 0\}$ is a subspace of R^3

c) $\{(a,ab) \mid a,b \in R\}$ is a subspace of R^2 d) $\{(a, 3a+b, -b) \mid a,b \geq 0\}$ is a subspace of R^3.

(v) The values of k which make the system $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & k \\ -3 & -3 & k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$ has unique solution

a) $k \neq -3$ b) $k \neq -1$ c) $k = -1$ d) None is correct.

(vi) Let $F = \{(a-b-2c,\ 3a-3b-6c,\ -2a+2b+4c,\ 3b-2c) \mid a,b,c \in R\}$. We know that F is a subspace of R^4. Then $dim(F) =$

a) 2 b) 3 c) 1 d) 4

(vii) One of the following is a basis for P_3

a) $\{1+x+x^2, -1-x-2x^2, 1+x+5x^2\}$ $\{3, 2x-x^2, 6+4x-2x^2\}$ c) $\{2x+3x^2, x^2, 4+x^2\}$ d) None of the previous is correct

(viii) Let $F = \{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \mid a_1 + a_4 = 0$ and $a_2 + a_3 = 0\}$. We know that F is a subspace of $R_{2\times2}$. Then $F =$

a) $span\{\begin{bmatrix} 4 & 2 \\ -2 & -4 \end{bmatrix}\}$ b) $span\{\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\}$ c) $span\{\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\}$ d) None is correct

(ix) Let A be a 2×2 matrix such that $[A \mid I_2]$ is row-equivalent to $\begin{bmatrix} 2 & -3 & \mid & 3 & 0 \\ 4 & 3 & \mid & -4 & 2 \end{bmatrix}$. Then $det(A) =$

a) -6 b) 18 c) 3 d) -1 e) 104

(x) Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & 4 \\ 2 & 2 & 2 & 2 \end{bmatrix}$ and let $T : R^4 \to R^3$ such that $T(a,b,c,d) = A \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$. Then Range(T) =

a)$Span\{(1,1,1,1),(-1,-1,-1,4)\}$ b) $Span\{(1,5,0),(1,0,0)\}$

c) $Span\{(1,-1,2),(1,4,2)\}$ d) None is correct

(xi) Let A and T as above. One of the following points lies in the Ker(T):

a) (1, 1, 0, 0) b) (0, 1, -1, 0) c) (1, -1, 0) d) (1, 0, 1, 0)

(xii) Let A and T as above. One of the following points lies in the range of T:

a)(5, -8, 0, 0) b) (0, 5, -5) c) (5,0, 0) d) (5, 0, -10)

(xiii) The values of k which make the system $\begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & k \\ -3 & -3 & k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$ consistent are

a) -3 b) -4 c) $k \neq -1$ and $k \neq -3$ d) -5 e) None of the previous.

(xiv) Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$. The eigenvalues of A are :

a)1 b) 0, 1, 2 c) 0, 1 d) None is correct

(xv) Let C, D be 3×4 matrices, A be an invertible matrix 4×4 and B be a 3×4 matrix such that $CA = DA = B$. Then

a) C must equal to D b) C does not need to equal to D but $Rank(C) = Rank(D)$ c) C does not need to equal to D d) Rank(C) = Rank(D) = 3 e) None of the previous is correct.

(xvi) Let $A^{-1} = \begin{bmatrix} 4 & 2 & -2 \\ -4 & -4 & 0 \\ 2 & 1 & -0.5 \end{bmatrix}$ Then the solution to the system $AX = \begin{bmatrix} 8 \\ -12 \\ 4 \end{bmatrix}$ is

a) $x_1 = 1, x_2 = 2, x_3 = 0$ b) $x_1 = 1, x_2 = 1, x_3 = -1$ c) $x_1 = -1, x_2 = 6, x_3 = 0$ d) None is correct

(xvii) Let A^{-1} as above. Then (1, 3)-entry of A is

a) 1 b) -1 c) 2 d) 0.5 e) None is correct

(xviii) If A, B are 3×3 matrices such that $det(2A) = det(B^{-1}) = 4$. Then $det(ABA^T) =$

a) 1 b) 1/4 c) 1/16 d) 64 e) none of the above

(xix) Given A is 2×2 and $A \ \overrightarrow{2R_2 + R_1 \to R_1} \ B$. Let E be an elementary matrix such that $EB = A$. Then $E =$

a) $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ d) None is correct

(xx) Given v_1, v_2, v_3 are independent points in R^7. One of the following statements is correct:

a) $v_1, v_2, 2v_1 + v_2$ are independent points in R^7 b) $v_1, -v_1 + v_3, 3v_2$ are independent points in R^7 c) $v_1, v_1 + v_2 + v_3, -4v_1 + v_2 + v_3$ are independent points in R^7 d) (b) and (c) are correct e) All previous statements are correct.

(xxi) Let $A \begin{bmatrix} 8 & 9 & -2 \\ 0 & b & 0 \\ -4 & -b & 1 \end{bmatrix}$ such that $AX = \begin{bmatrix} 8 \\ 0 \\ -4 \end{bmatrix}$ has infinitely many solution. Then $b =$

a) 0 b) -9 c)9 d) any real number e) None is correct

(xxii) Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$ Given $x_1 = 2, x_2 = 1, x_3 = 0$ is a solution to the system $AX = \begin{bmatrix} 3a_3 \\ 3a_6 \\ 3a_9 \end{bmatrix}$. Then

a) $det(A) = 0$ b) The system $AX = \begin{bmatrix} 3a_3 \\ 3a_6 \\ 3a_9 \end{bmatrix}$ has infinitely many solutions c)$Rank(A) \leq 2$

d) (a) and (b) and (c) are correct e) None of the previous is correct

(xxiii) Let A be a 4×4 matrix such that $det(A) = -4$. Let B be the fourth column of A. Then one of the following statement is correct:

a)It is possible that the system has infinitely many solutions and $x_1 = x_2 = x_3 = 5/12$, $x_4 = -1/4$ is a solution to the system $AX = B$.

b) It is possible that the system $AX = B$ has infinitely many solutions and $x_1 = x_2 = x_3 = 5/4$, $x_4 = -1/4$ is a solution to the system $AX = B$.

c)$x_1 = x_2 = x_3 = 5/12$, $x_4 = -1/4$ is the only solution to the system $AX = B$.

d) $x_1 = x_2 = x_3 = 0$, $x_4 = 1$ is the only solution to the system $AX = B$.

QUESTION 2. (**6 points**)

Let $F = span\{(1,0,1,0),(0,1,1,0),(0,0,1,1)\}$ Find an orthogonal basis for F

QUESTION 3. (**15 points**) Given $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ is diagnolizable.

1)(**6 points)** Find all eigen-spaces of A and write each one as a span of a basis.

2) (**3 points)** Find an invertible matrix Q and a diagonal matrix D such that $Q^{-1}AQ = D$.

3)(**3 points)** For the Q you calculated in (2) find Q^{-1}.

4)(**3 points)** Find an invertible matrix M such that $MA = D$ where D is the matrix you calculated in (2).

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